$\mathcal{L}_1$ Theory

The architectures of $\mathcal{L}_1$ adaptive control theory decouple estimation (adaptation) from control (robustness), and with that allow for arbitrary FAST adaptation rates with bounded away from zero time-delay margin. The architectures of this theory, with a single set of control design parameters and without any redesign or returning, lead to uniform (a priori predictable) performance bounds throughout the entire operation of the system, including both transient and steady-state phases. The proofs of stability and performance use the $\mathcal{L}_1$-norm of a cascaded system for determining the bandwidth of the filter, used in the design. Because the $\mathcal{L}_1$-norm is the induced $\mathcal{L}_\infty$-norm of the input/output signals, the sufficient condition for stability, written in terms of $\mathcal{L}_1$-norm, leads to uniform performance bounds for input/output signals. The architectures of $\mathcal{L}_1$ adaptive control theory aim for (partial) compensation of uncertainties within the bandwidth of the control channel. The benefit of $\mathcal{L}_1$ architecture is that it does not invert the desired system dynamics. Instead, the fast estimation loop in $\mathcal{L}_1$ adaptive control architectures approximates the inverse map. This consequently allows for application of $\mathcal{L}_1$ adaptive controllers to a broader class of systems in the presence of various nonlinearities.

The architectures of $\mathcal{L}_1$ adaptive control theory are synthesized systematically, using three main elements: state predictor, estimation laws and a filtering structure for synthesis of the control law. The state predictor and the estimation laws reconstruct the estimates of uncertainties, while the low-pass filtering structure compensates for these estimates of the uncertainties within the bandwidth of the control channel. A typical architecture has the following structure:

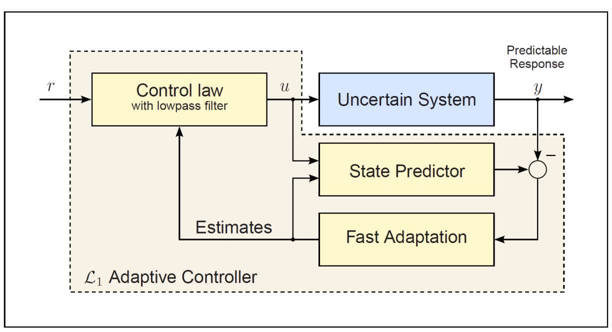

Figure 1: Generic $\mathcal{L}_1$ Adaptive Control Architecture

Subject to a certain set of assumptions, the input/output signals of this generic $\mathcal{L}_1$ adaptive control architecture approximate the same signals of the reference non-adaptive architecture

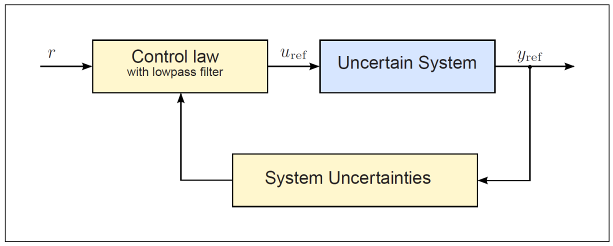

Figure 2: Reference Architecture

The closeness of the input/output signals of the adaptive and non-adaptive architectures can be quantified according to the following performance bounds:

$$|y - y_{\mathrm{ref}}|_{\mathcal{L}_\infty} \leq \frac{\gamma_1}{\sqrt{\Gamma}}, \quad |u - u_{\mathrm{ref}}|_{\mathcal{L}_\infty} \leq \frac{\gamma_2}{\sqrt{\Gamma}},$$

where γ_1 and γ_2 are positive constants, while Γ is the estimation rate.