MECHANICAL SCIENCE AND ENGINEERING
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$\mathcal{L}_1$- $\mathcal{GP}$: $\mathcal{L}_1$ Adaptive Control with Bayesian Learning

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Introduction

We present $\mathcal{L}_1$-$\mathcal{GP}$, an architecture based on $\mathcal{L}_1$ adaptive control and Gaussian Process Regression (GPR) for ***safe simultaneous control and learning***.

- GPR allows data-efficient learning of non-parametric uncertainties.
- $\mathcal{L}_1$ adaptive control provides stability and transient performance guarantees, which allows for GPR to efficiently and safely learn model uncertainties.
- Learned dynamics can be seamlessly integrated into the $\mathcal{L}_1$ adaptive control architecture.
- Learned dynamics can be further used for improved performance and/or robustness.

Manuscript can be found at [1].

Problem Setup

Dynamics:

$$\dot{x}(t) = A_m x(t) + B_m(u(t) + f(x(t))), \quad x(0) = x_0$$
$$y(t) = C_m x(t).$$

- The model uncertainty $f(x)$ is learned via GPR.
- The matrices A_m, B_m, C_m are known and define the *ideal* performance.
- Control input $u(t)$ attempts to compensate for $f(x)$ while tracking a reference signal.

The Components

Bayesian Learning via GPR

- Data-efficient non-parametric approach to regression.
- Natural notion of uncertainty quantification.
- Uncertainty quantification allows the computation of uniform prediction error bounds [2].
- Prediction error bounds act as certificates of quality of learned estimates.

$\mathcal{L}_1$ Adaptive Control

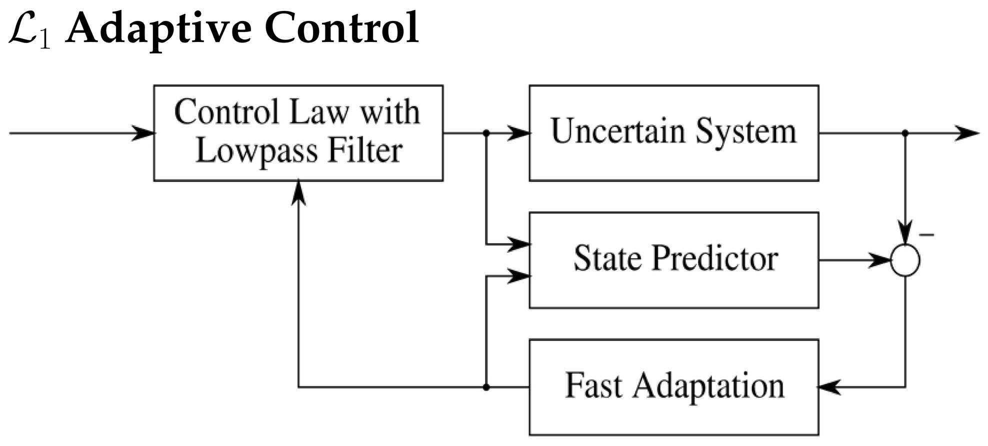

Figure 1 : $\mathcal{L}_1$ adaptive control architecture

- Robust adaptive control with quantifiable uniform bounds in *both* transients and steady state.
- Estimation is decoupled from control via the low-pass filter.
- Arbitrary fast estimation is allowed while maintaining desired robustness margins.

The $\mathcal{L}_1$-$\mathcal{GP}$ Architecture

A symbiotic combination of GPR and $\mathcal{L}_1$ adaptive control.

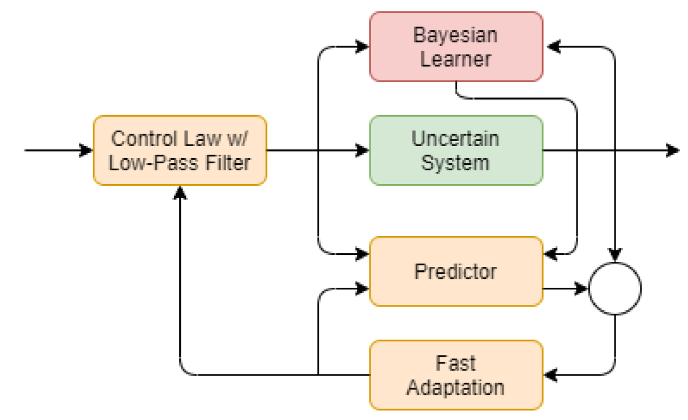

Figure 2 : The $\mathcal{L}_1$-$\mathcal{GP}$ Architecture

- The $\mathcal{L}_1$ architecture allows GPR estimates to be safely incorporated via the predictor.
  - Ensures performance and robustness during learning transients.
  - Fast adaptive element compensates for the initial poor performance of learning.
- GPR learns model uncertainties without persistently exciting signals.
  - Uses stored data via the kernel matrix.
  - Estimates improve whenever excitation is available.
  - Parametric models of uncertainties are not required.
  - Any prior knowledge, if available, can be directly incorporated via the kernel function.

Simulation Results

- Example: Angular rate control of a 3-D quadrotor.
- Setup for model learning:
  - Data collection rate: 1 Hz.
  - Model update rate: 0.1 Hz.

Simulation Scenario 1

We consider static state-dependent uncertainties.

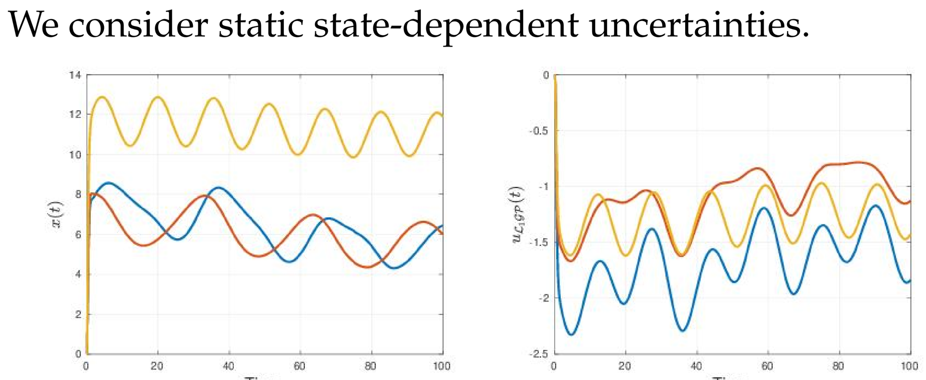

Figure 3 : Evolution of states (left) and control inputs (right).

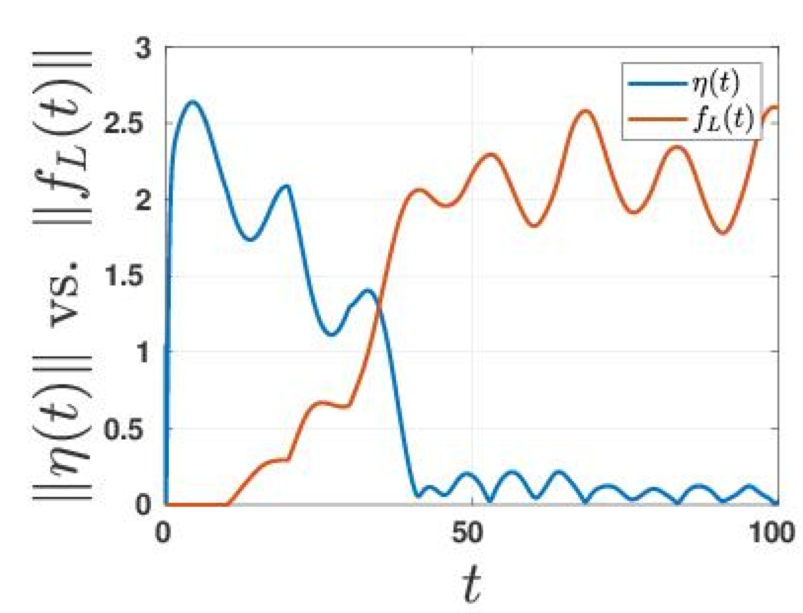

Figure 4 : Control contribution evolution. Fast adaptive element (blue) vs. GPR estimate (orange)

- In the absence of sufficient data, the $\mathcal{L}_1$ adaptive element (blue) is the primary contributor to the total control input.
- As learning improves, the control contribution transitions to the GPR estimate (orange).

Simulation Scenario 2

We consider state-dependent uncertainties which switch at $t = 35$ s.

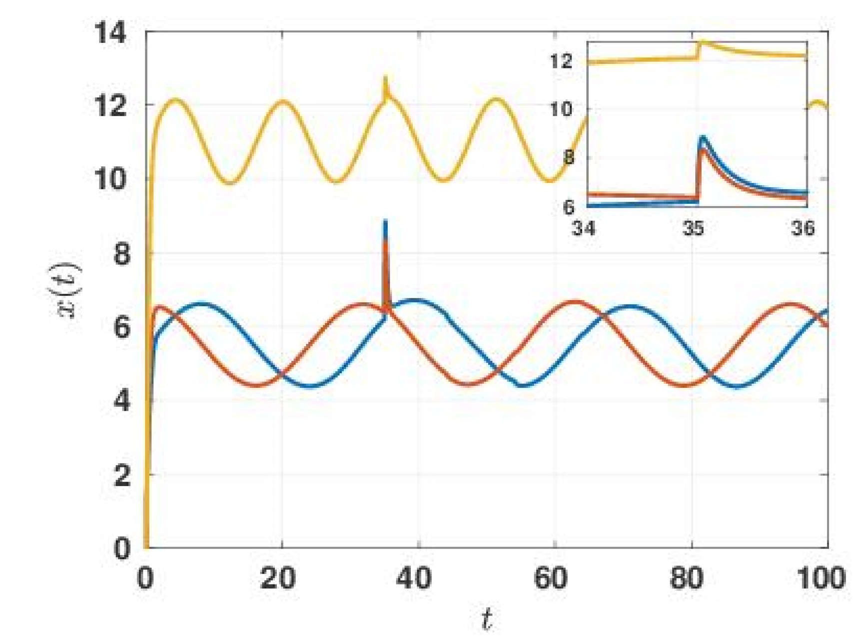

Figure 5 : State evolution across the uncertainty switch.

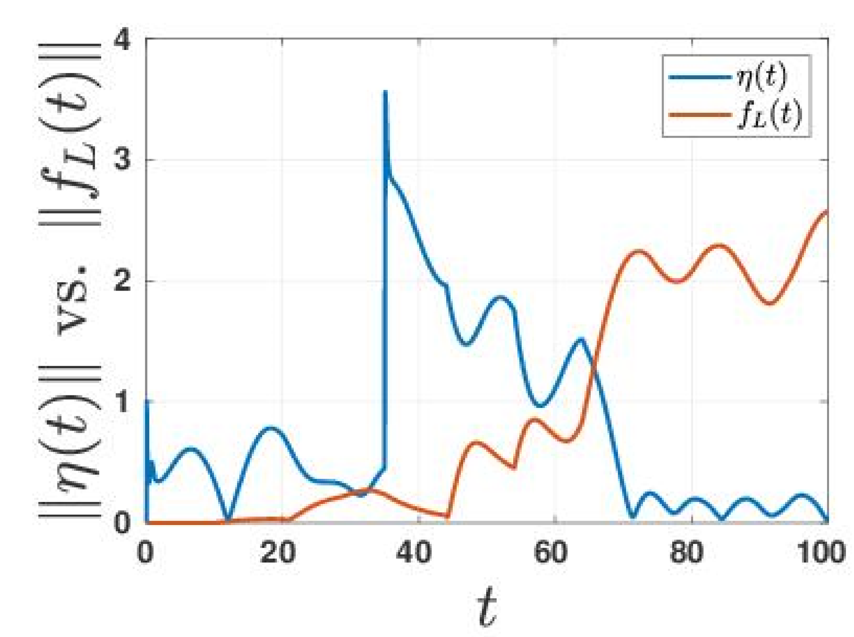

Figure 6 : Control contribution evolution. Fast adaptive element (blue) vs. GPR estimate (orange)

- When the model uncertainties switch, $\mathcal{L}_1$ adaptive element immediately steps in to compensate for the new unlearned uncertainties.
- The transition from the $\mathcal{L}_1$ adaptive element (blue) to the GPR estimate (orange) is again observed as learning catches up.

Future Work

Future work will extend the safe-learning methods to a larger class of systems by

- guaranteeing tracking error bounds during learning for nonlinear systems subject to spatio-temporal disturbances;
- demonstrating improved performance while ensuring safety by incorporating learned dynamics into trajectory generation methods.

References

[1] A. Gahlawat, P. Zhao, A. Patterson, N. Hovakimyan, and E. A. Theodorou, "$\mathcal{L}_1$-$\mathcal{GP}$: $\mathcal{L}_1$ adaptive control with Bayesian learning," *arXiv preprint arXiv:2004.14594*, 2020.

[2] A. Lederer, J. Umlauft, and S. Hirche, "Uniform error bounds for Gaussian process regression with application to safe control," in *Advances in Neural Information Processing Systems*, 2019, pp. 657–667.

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