**L₁- GP: L₁ Adaptive Control with Bayesian Learning**

**Introduction**

We present L₁-GP, an architecture based on L₁ adaptive control and Gaussian Process Regression (GPR) for safe simultaneous control and learning:

- **L₁** adaptive control allows data-efficient learning of non-parametric uncertainties.
- **L₁** adaptive control provides stability and transient performance guarantees, which allows for GPR to efficiently and safely learn model uncertainties.
- Learned dynamics can be seamlessly integrated into the L₁ adaptive control architecture.
- Learned dynamics can be further used for improved performance and/or robustness.

Manuscript can be found at [1].

**Problem Setup**

Dynamics:

\[
\begin{align*}
\dot{x}(t) &= A_0 x(t) + B_0 u(t) + f(x(t)), \\
y(t) &= C_m x(t),
\end{align*}
\]

- The model uncertainty \( f(x) \) is learned via GPR.
- The matrices \( A_0, B_0, C_m \) are known and define the ideal performance.
- Control input \( u(t) \) attempts to compensate for \( f(x) \) while tracking a reference signal.

**The Components**

**Bayesian Learning via GPR**

- Data-efficient non-parametric approach to regression.
- Natural notion of uncertainty quantification.
- Uncertainty quantification allows the computation of uniform prediction error bounds [2].
- Prediction error bounds act as certificates of quality of learned estimates.

**L₁ Adaptive Control**

\[
\begin{align*}
\text{Control Law with} & \quad \text{Uncertain System} \\
\text{Lowpass Filter} & \quad \text{State Predictor} \\
& \quad \text{Fast Adaptation}
\end{align*}
\]

- Robust adaptive control with quantifiable uniform bounds in both transients and steady state.
- Estimation is decoupled from control via the low-pass filter.
- Arbitrary fast estimation is allowed while maintaining desired robustness margins.

**The L₁-GP Architecture**

A symbiotic combination of GPR and L₁ adaptive control.

**Simulation Results**

- Example: Angular rate control of a 3-D quadrotor.
- Setup for model learning:
  - Data collection rate: 1 Hz.
  - Model update rate: 0.1 Hz.

**Simulation Scenario 1**

We consider static state-dependent uncertainties.

**Future Work**

Future work will extend the safe-learning methods to a larger class of systems by:

- Guaranteeing tracking error bounds during learning for nonlinear systems subject to spatio-temporal disturbances.
- Demonstrating improved performance while ensuring safety by incorporating learned dynamics into trajectory generation methods.

**References**
